shortcoming, since in this case we have neither a diffracted wave, nor a penumbral zone. In this case we cannot obtain any diffusion equations within the focusing zone. These two limiting cases, namely the case of a small aperture angle of the reflector discussed in /6/, and a fully opened reflector, lead to basically different structures of the solution within the focusing zone. We can obtain a solution suitable for all cases only by taking into account the penumbral zone away from the focus. We note that the methods used in /10/ do not enable the effect of the reflector edges on the focusing of the shock wave in a viscous fluid to be taken into account, since they are based on the wave acoustics of ideal media.

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APPROXIMATE FORMULAS FOR HEAT FLOWS TOWARDS AN IDEALLY CATALYTIC SURFACE NEAR A PLANE OF SYMMETRY*

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The three-dimensional flow of a chemically unstable viscous gas near a plane of symmetry of blunt bodies streamlined at the angle of attack, is considered. The investigation is carried out using a model of a thin, viscous shock layer. To a first approximation of the method of successive approximations for a uniform gas simple formulas are obtained for the distribution of the heat flux over the surface, referred to its value at the stagnation point. It is shown that for a chemically unstable gas the distribution of the heat flux along an ideally catalytic surface depends only slightly on the conditions prevailing within the incident flow, is determined mainly by the geometrical characteristics of the body, and is described quite satisfactorily by the formulas obtained. The accuracy of these formulas is determined by comparison with numerical computations carried out for bodies of various shapes, moving at different angles of attack along a planing trajectory of re-entry into the Earth's atmosphere, and during re-entry into the atmosphere at a constant velocity.

1. Let us consider the three-dimensional steady flow past a blunt body of a stream of

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viscous, chemically unstable gas at a very high supersonic velocity, with the flow pattern changing from the "diffused" layer mode in which the viscosity is appreciable over the whole region of perturbed flow, to modes with a sharply delineated boundary layer. Let $z = f(x, y)$ be the equation of the surface of the body in a Cartesian system of coordinates, let the velocity vector V_∞ of the incident flow coincide in direction with the z axis and let the origin of coordinates be at the stagnation point. We choose a system of curvilinear, non-orthogonal coordinates (x^1, x^2, x^3), attached normally to the streamlined surface, $x^3 = \text{const}$ is a family of surfaces parallel to the surface of the body $(x^3 = 0)$, and we choose x^1 and x^2 at the surface as follows: $x^1 = x$, $x^2 = y$, $z = f(x^1, x^2)$.

Let $x^2 = 0$ be the plane of symmetry of the body. We shall study the flow in the neighbourhood of this surface using a model of a thin, three-dimensional viscous shock layer /1/. Taking into account the unstable chemical reactions and the multicomponent diffusion, while neglecting thermal and barodiffusion, we can write the equations of the viscous shock layer near the plane of symmetry in the form

$$\frac{\partial}{\partial x^1}(\rho u^1) + \rho u^2 \sqrt{a} + \frac{\partial}{\partial x^3}(\rho u^3 \sqrt{a}) = 0, \quad \rho D u^1 = -\frac{x^1}{\sqrt{a}} p_1 + \frac{\partial}{\partial x^3}\left(\frac{\mu}{\mathrm{Re}_\infty}\frac{\partial u^1}{\partial x^3}\right) \tag{1.1}$$

$$\rho\left(Du^2 + (u^2)^2 + A_1 (u^1)^2\right) = -p_2 + A_2 \frac{\partial p}{\partial x^1} + \frac{\partial}{\partial x^3}\left(\frac{\mu}{\mathrm{Re}_\infty}\frac{\partial u^2}{\partial x^3}\right)$$

$$\frac{\partial p}{\partial x^3} = \rho \frac{b}{a\sqrt{a}}(u^1)^2, \quad \frac{\partial p_1}{\partial x^3} = \frac{1}{x^1}\frac{\partial}{\partial x^1}\left(\rho \frac{b}{a\sqrt{a}}(u^1)^2\right)$$

$$\frac{\partial p_2}{\partial x^3} = \rho\left[\left(f_{1122}^{\mathrm{IV}} - \frac{f_{11}''}{a}\left(f_1' f_{122}''' + (f_{22}'')^2\right)\right)\frac{(u^1)^2}{a\sqrt{a}} + 4\frac{f_{122}'''}{a} u^1 u^2 + \frac{2 f_{22}''}{\sqrt{a}}(u^2)^2\right]$$

$$\rho c_p DT = \frac{\partial}{\partial x^3}\left(\frac{\mu c_p}{\mathrm{Re}_\infty \sigma}\frac{\partial T}{\partial x^3}\right) + \frac{\mu}{\mathrm{Re}_\infty}\left(\frac{\partial u^1}{\partial x^3}\right)^2 - \sum_{i=1}^{N} h_i w_i^{\cdot} - \left(\sum_{i=1}^{N} c_{pi} I_i\right)\frac{\partial T}{\partial x^3} + \frac{u^1}{\sqrt{a}}\frac{\partial p}{\partial x^1}$$

$$\rho D c_i + \frac{\partial I_i}{\partial x^3} = w_i, \quad i = 1, \ldots, N - \mathrm{Ne}$$

$$\rho D c_k^* + \frac{\partial I_k^*}{\partial x^3} = 0, \quad k = 1, \ldots, \mathrm{Ne} - 1$$

$$\frac{\mu}{\mathrm{Re}_\infty}\frac{\partial m c_i}{\partial x^3} = \sum_{j=1}^{N}\frac{m^2}{m_j} S_{ij}(c_i I_j - c_j I_i), \quad i = 1, \ldots, N - 1$$

$$p = R_G \rho \frac{T}{m}, \quad \sum_{k=1}^{\mathrm{Ne}} I_k^* = 0, \quad \sum_{k=1}^{\mathrm{Ne}} c_k^* = 1, \quad m = \left(\sum_{i=1}^{N}\frac{c_i}{m_i}\right)^{-1}$$

$$\mathrm{Re}_\infty = \frac{\rho_\infty V_\infty R}{\mu_\infty}, \quad c_p = \sum_{i=1}^{N} c_i c_{pi}, \quad h = \sum_{i=1}^{N} c_i h_i$$

$$p_\alpha = \frac{1}{x^{(\alpha)}}\frac{\partial p}{\partial x^\alpha}, \quad \alpha = 1, 2, \quad D = \frac{u^1}{\sqrt{a}}\frac{\partial}{\partial x^1} + \frac{u^3 \partial}{\partial x^3}$$

$$a = 1 + f_1'^2, \quad b = f_{11}'', \quad A_1 = \frac{1}{a^2} f_{11}'' f_{22}'', \quad A_2 = \frac{1}{a} f_1' f_{22}''$$

$$f_1' = \frac{\partial f}{\partial x^1}, \quad f_2' = \frac{\partial f}{\partial x^2}, \quad f_{11}'' = \frac{\partial^2 f}{\partial (x^1)^2}, \quad f_{22}'' = \frac{\partial^2 f}{\partial (x^2)^2}$$

Here $V_\infty u^1$, $V_\infty x^2 u^2$, $V_\infty u^3$ are the physical components of the velocity vector corresponding to the axes x^1, x^2, x^3; $\rho_\infty V_\infty^2 p$, $\rho_\infty \rho$, $T_0 T$ are the pressure, density and temperature, respectively, of the mixture of gases containing N chemical components $(T_0 = V_\infty^2 / c_{p\infty})$; $\mu_\infty \mu$, $c_{p\infty} c_p$, Re_∞, σ, m are the coefficient of viscosity, specific heat capacity, Reynolds number, Prandtl number and molecular mass of the mixture c_i, m_i, $c_{p\infty} T_0 h_i$, $c_{p\infty} c_{pi}$, $\rho_\infty V_\infty I_i$, $\rho_\infty V_\infty w_i^{\cdot}/R$ is the mass concentration, molecular mass, specific enthalpy and heat capacity, the normal component of the diffusion flux vector of the k-th element, Ne is the number of elements, S_{ij} are the binary Schmidt numbers, $c_{p\infty} R_G$ is the universal gas constant and V_∞ is the modulus of the velocity vector of the incoming flow. All linear dimensions are referred to the characteristic linear dimension R representing the radius of curvature of the surface at the stagnation point in the plane of symmetry. The indices ∞, w, s refer, respectively, to the parameters in the incoming flow, at the surface of the body, and at the inner boundary of the shock wave.

When specifying the bounary conditions on the surface of an impermeable body, which is assumed to be an uniform emitter, we take into account the effect of the catalytic recombination of atoms on the wall, as well as the rate of slippage and the temperature jump in the multicomponent, unstable gas mixture /2, 3/:

$$u^\alpha = \frac{2-\theta}{\theta}\sqrt{\frac{\pi}{2R_GT}}\frac{1}{\mathrm{Re}_\infty}\frac{\mu}{\rho}\left(\sum_{i=1}^{N}\frac{c_i}{\sqrt{m_i}}\right)^{-1}\frac{\partial u^\alpha}{\partial x^3}, \quad u^3 = 0 \tag{1.2}$$

$$\frac{T}{T_w} = \left(1 + \frac{2-\theta}{\theta}\frac{1}{E}\sum_{i=1}^{N}\frac{I_i}{m_i}\right)\left(1 - \frac{2-\theta}{\theta}\frac{c_p}{\sigma}\frac{1}{\mathrm{Re}_\infty}\frac{\mu}{2R_GET}\frac{\partial T}{\partial x^3}\right)^{-1}$$

$$E = \rho\sqrt{\frac{2R_GT}{\pi}}\sum_{i=1}^{N}\frac{c_i}{m_i^{3/2}}, \quad q = \frac{\varepsilon\sigma_B T_0^4}{\rho_\infty V_\infty^3}T_w^4$$

$$q = \frac{\mu c_p}{\sigma\,\mathrm{Re}_\infty}\frac{\partial T}{\partial x^3} + \frac{\mu}{\mathrm{Re}_\infty}u^1\frac{\partial u^1}{\partial x^3} - \sum_{i=1}^{N}h_iI_i$$

$$I_i = -\frac{2}{2-\gamma_i}k_{wi}\sqrt{\frac{T}{T_w}}\rho c_i, \quad k_{wi} = \gamma_i(T_w)\sqrt{\frac{R_GT_w}{2\pi m_i}}, \quad i = 1, \ldots, N-\mathrm{Ne}$$

$$I_j^* = 0, \; j = 1, \ldots, \mathrm{Ne}-1$$

Here θ, γ_i, ε are the accommodation coefficient, the catalytic recombination coefficient and the surface blackness coefficient, and σ_B is the Stefan-Boltzmann constant.

We specify on the shock wave the generalized Rankine-Hugoniot relations which, in the approximation of high supersonic velocities neglecting chemical reactions, take the following form /4/:

$$\rho\left(u^3 - \frac{u^1}{\sqrt{a}}\frac{\partial x_s^3}{\partial x^1}\right) = u_\infty^3, \quad u_\infty^3(u^\alpha - u_\infty^\alpha) = \frac{\mu}{\mathrm{Re}_\infty}\frac{\partial u^\alpha}{\partial x^3}, \quad \alpha = 1, 2 \tag{1.3}$$

$$u_\infty^3\left(H - H_\infty - \frac{(u_\infty^3)^2}{2}\right) = \frac{\mu c_p}{\sigma\,\mathrm{Re}_\infty}\frac{\partial T}{\partial x^3} - \sum_{i=1}^{N}h_iI_i + \frac{\mu}{\mathrm{Re}_\infty}u^1\frac{\partial u^1}{\partial x^3}$$

$$p = (u_\infty^3)^2, \; H = h + 0{,}5\,(u^1)^2$$

$$u_\infty^3(c_k^* - c_{k\infty}^*) + I_k^* = 0, \; k = 1, \ldots, \mathrm{Ne}-1$$

$$u_\infty^3(c_i - c_{i\infty}) + I_i = 0, \; i = 1, \ldots, N-\mathrm{Ne}$$

When considering the chemical reactions, we assume that the following five components are present in the perturbed region of the flow: N_2, O_2, NO, N, O, and that dissociation-recombination and exchange reactions take plane between them. The system of reactions, the reaction rate constants and the transport coefficients are identical with those used in /5/, and we assume that the internal degrees of freedom are equally excited.

2. The equations of a thin, viscous shock layer in a homogeneous gas near the plane of symmetry of a blunt body with a given surface temperature were solved using the integral method of successive approximations /6, 7/. The equations of momentum and energy were integrated twice over the transverse coordinate, taking the boundary conditions into account. In order to solve the resulting system of integrodifferential equations, we constructed an iterative process in which every consecutive approximation is expressed, for the functions sought, in terms of the integrals of the functions of the preceding approximation. In order that all the approximations should satisfy the boundary conditions on the body as well as on the shock wave, we introduced, at every step of the iterative process, additional control functions $\Delta_i(x)$ $(i = u^1, u^2, H)$, for which we obtained ordinary differential equations which do not, in general, have analytic solutions.

Earlier /8/ the first approximation of this method was used with the zero-th approximations for the velocity components and enthalpy in the form of linear functions to obtain the solution of the problem for the case when the equations for the control functions were solved in the locally selfsimilar approximation, i.e. neglecting terms containing the derivatives $d\Delta_i/dx$. For moderate and high Reynolds numbers $(\mathrm{Re}_\infty \geqslant 1000)$, when the slippage effects at the body and on the shock wave can be neglected, we can simplify the equations for Δ_i, integrate them, and thus obtain a non-locally selfsimilar analytic solution in the first approximation.

In particular we obtain for the distribution of the Stanton number $c_H = q/\rho_\infty V_\infty(H_\infty - H_w)$ along the stream line,

$$c_H = \frac{\cos^2\alpha}{6\lambda I\Delta_H}, \quad I = \frac{(\gamma-1)}{27\gamma}\mathrm{Re}\,\sigma, \quad \lambda = 1 + \frac{4}{15}\frac{\mathrm{tg}^2\alpha H^*}{H} \tag{2.1}$$

$$\operatorname{tg}^2\alpha\Delta_H\frac{d\Delta_H}{dx}+\frac{2H}{\cos\alpha}\Delta_H{}^2=\frac{1}{\lambda I}$$

Here γ is the ratio of the specific heats, Re is the Reynolds number behind the shock wave, α is the angle between the incoming flow and the normal to the surface, H^* is the curvature of the stream line near the plane of symmetry, and H is the mean curvature of the surface at the given point, equal to half the sum of the principal curvatures. In the chosen coordinate system we have

$$\operatorname{tg}\alpha=f_1',\quad H^*=\frac{f_{11}''}{(1+f_1'^2)^{3/2}},\quad H=\frac{1}{2}\left(H^*+\frac{f_{22}''}{\sqrt{1+f_1'^2}}\right)$$

Integrating the second equation of (2.1) we obtain from the first equation the following relation for the magnitude of the heat flux referred to its value at the stagnation point:

$$q_*=\frac{q}{q_0}=\frac{\cos^2\alpha F^{1/2}}{2}\left[H_0\lambda\int_0^s\frac{F\cos\alpha ds}{\lambda\operatorname{tg}\alpha}\right]^{-1/2},\quad F=\exp\int_0^s\frac{4H}{\operatorname{tg}\alpha}ds \tag{2.2}$$

Here the subscript 0 denotes quantities at the stagnation point, and s is the arc length measured from this point $(dx=\cos\alpha ds)$.

Solving the second equation of (2.1) in the locally selfsimilar approximation, we obtain

$$q_*=\sqrt{H\cos^3\alpha/H_0\lambda} \tag{2.3}$$

This formula differs from the analogous formula obtained in /8/ in having the multiplying factor λ^{-1}. It is due to the deviation of the pressure distribution from the Newtonian distribution, and the formula is therefore more accurate.

From the analytic solution obtained it follows that at moderate and high Reynolds numbers (Re $\geqslant 100$) the magnitude of relative heat flux is independent of Re, as well as of σ, γ and the surface temperature T_w (for a cooled wall). The fact that q_* ceases to depend on Re and depends only slightly on the remaining parameters characterizing the flow of a uniform gas, is determined mainly by the geometrical characteristics of the streamlined body, and can be described quite satisfactorily by the formulas obtained, is fully confirmed by numerical computations carried out over a wide range of variation of these parameters (Re $=10^2...10^4$, $\gamma=1.1...1.67$, $T_w=0...0.5$).

Comparison with numerical results obtained for bodies of various shapes streamlined at various angles of attack (from 0 to 45°) has shown that formulas (2.2) are accurate in the case of a homogeneous gas. The computations showed that the difference between the results obtained using formulas (2.2) and (2.3) does not exceed, in the majority of cases, several percent. However, formula (2.2) gives a much greater accuracy in the case of a flow at the angle of attack from the side of the stagnation point at which the radius of the longitudinal curvature of the contour of the body decreases.

Note that the formulas obtained for q_* can also be used for axisymmetric flows. In this case we have

$$q_*=\frac{\cos^2\alpha\sin\alpha r}{2}\left[H\lambda\int_0^s\frac{\cos^4\alpha r^2}{\lambda\sin\alpha}ds\right]^{-1/2}$$

Here r is the distance between the surface of the body and the axis of symmetry, R is the radius of curvature of the generatrix, and $H=0.5\,(1/R+\sin\alpha/r)$.

Investigations carried out show that for an ideally catalytic surface the distribution of the relative heat flux is independent of the way in which the chemical reactions proceed within the shock layer, and is close to the distribution which occurs in a flow of homogeneous gas. Formulas (2.2) and (2.3) hold with a sufficient degree of accuracy for values of the relative heat flux and for the chemically unstable flows in the case of an ideally catalytic surface, and this is true not only for the given surface temperature, but also for the case when the temperature is determined from the condition for equilibrium irradiation of the wall.

In order to check the validity and assess the accuracy of formulas (2.2) and (2.3) for chemically unstable flows, we solved system (1.1) with boundary conditions (1.2), (1.3) numerically, using a method analogous to that given in /5/. We used the scheme in /9/ with fourth order of accuracy of approximation over the transverse coordinate. At heights of $h\leqslant 75$ km the mesh was tightened in the region near the shock wave and near the body.

The distribution of the relative heat fluxes over the surface obtained using formulas (2.2) and (2.3), was compared with the numerical solution of Eqs.(1.1) for various elliptic paraboloids, two-sheet hyperboloids and triaxial ellipsoids streamlined at angles of attack ranging from 0 to 45°. The conditions in the incoming flow corresponding, firstly, to motion at altitudes of 100 to 50 km over a planing trajectory of re-entry into the Earth's atmosphere /10/ which was assumed to be isothermal, with a density distribution depending on the altitude h(km) : $\rho_\infty = 1.225 \cdot 10^{-8} \exp \cdot (-0.142h)$g/cm^3 , $T_\infty = 200°$K and, secondly, to motions at the same heights at a constant velocity of $V_\infty = 8$ km/sec. Figs.1-4 show some results of a comparison of the approximate and exact solutions (we assumed, in the course of the computation, that $\varepsilon = 0.85$, $\theta = 1$, $R =$ 0.7 m).

Fig.1 shows the distribution of the relative heat flux along the stream line for a two-sheet hyperboloid with a half aperture angle of 40°, in the plane $y = 0$, and a ratio of the principal curvatures at the stagnation point $k = 2.5$ (curves a), and for an ellipsoid with a ratio of the squares of the semi-axes $1 : k : 0.5$; $k = 2.5$ (curve b) with angle of attack $\alpha = 0°$. The distributions of q_*, obtained from a numerical solution of Eqs.(1.1) for an ideally catalytic surface and the planning trajectory of re-entry within an altitude range of 60 to 90 km, lie within the shaded area, while the dark and light dots show the results obtained using formulas (2.2) and (2.3) respectively.

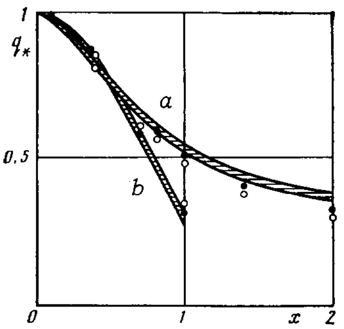

Fig.1

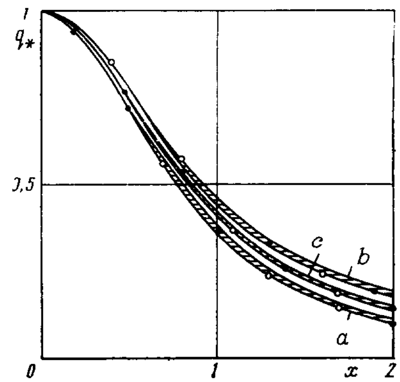

Fig.2

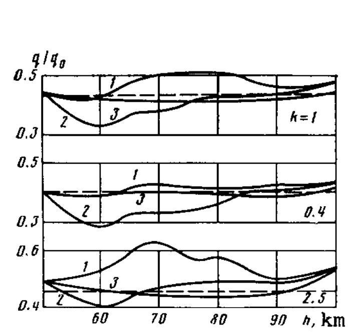

Fig.3

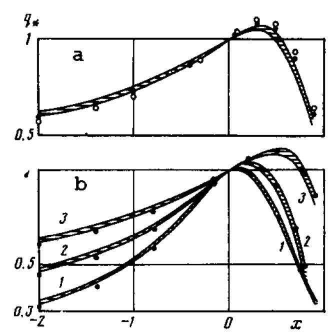

Fig.4

Fig.2 shows the distributions of the relative heat flux along the stream lines for various elliptic paraboloids streamlined at zero angle of attack, with a ratio of the principal curvatures at the stagnation point equal to $k = 0.4$ (curves a), $k = 2.5$ (b) and $k = 1$ (c is an axisymmetric paraboloid). Eqs.(1.1) were solved numerically for an ideally catalytic surface at altitudes of 50 to 90 km, for a planning re-entry trajectory, as well as for the trajectory of re-entry at a constant velocity. All distributions of q_* at these altitudes calculated for both trajectories using formulas (2.2) and (2.3), lie within the shaded areas.

Fig.3 shows how the value of q_* varies with the flight altitude h (planning trajectory) on a side surface at the point $x = 1$, for a surface with different catalytic properties. Results are given for elliptic paraboloids with $k = 1$; 0.4 and 2.5. Curves 1 show the numerical solution of Eqs.(1.1) for a non-catlaytic surface, curves 2 for a surface on which heterogeneous first-order reactions take place with rate constants which depend on temperature /11/, and curves 3 for an ideally catalytic surface, with dashed lines corresponding to results obtained using formulas (2.2) and (2.3) (they practically coincide). We see that for an ideally catalytic surface the magnitude of the relative heat flux on the side surface is practically independent of the flight altitude, i.e. of the conditions in the incoming flow, while for other models of catalytic properties of the surface such a dependence is essential.

In Fig.4 the analytic and numerical solutions are compared for bodies streamlined at the angle of attack. The distributions of q_* are given along the lines of flow for a hyperboloid with aperture half-angle $40°, k = 2.5$, and angle of attack $\alpha = 30°$ (Fig.4a), and an elliptic paraboloid for $k = 0.4$ $\alpha = 15, 30, 45°$ - curves *1-3* respectively (Fig.4b). The shaded regions correspond to numerical solutions of Eqs.(1.1) for an ideally catalytic surface (planning trajectory) at altitudes ranging from 50 to 90 km, with the dark and light dots representing the results obtained from formulas (2.2) and (2.3).

The results of the investigation have shown that the distribution of the relative heat flux on the ideally catalytic surface of a blunt body depends weakly on the degree of dissociation in the shock layer, including the frozen, chemically non-equilibrium and nearly equilibrium modes of flow, is basically determined by the shape of the body, and is satisfactorily described by the formulas given.

It should be noted that unlike the analogous forms for the values of q_*, proposed in boundary-layer theory (e.g. in /12, 13/), formulas (2.2) and (2.3) firstly do not require a knowledge of the parameters at the outer boundary of the boundary layer, i.e. of the solution of inviscid flow, and depend only on the geometrical characteristics of the streamlined body, and secondly they can be used not only at large but also at moderate Reynolds numbers.

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